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METHOD FOR MANAGING IN-FLIGHT REFUELLING OF A FLEET OF AIRCRAFT

Abstract: In-flight refuelling of a fleet (1) of aircraft by a refuelling aircraft (2) through a refuelling boom requires the formation of a queue, and the determination of a number, one or possibly two, of passes for each aircraft at the refuelling boom, thereby constituting a refuelling sequence. If the order of the queue is easy to establish, since it often corresponds to a classification in descending order of importance of the requests for fuel, the same is not true of the number of passes at the refuelling boom, which has to be reduced to a minimum while observing the flying ranges of the aircraft to be refuelled before starting their own refuelling. The inventive method, which can be implemented by a computer, enables automation of the search for a refuelling sequence fulfilling the requirements of the flying ranges of various aircraft being refuelled, while minimising the number of passes of the aircraft at the refuelling boom.

METHOD FOR MANAGING IN-FLIGHT REFUELLING OF A FLEET OF AIRCRAFT

The present invention concerns the in-flight refuelling of a fleet of aircraft, and in particular, the determination of the refuelling sequence most suitable to the current situation.

The current strategic context more and more commonly causes fleets of aircraft to be called into action far from their base, or on relatively long missions which require one or more in-flight refuelling operations. In-flight refuelling of a fleet of aircraft requires the organisation of a rendezvous between the fleet of aircraft to be refuelled and the refuelling aircraft, and the determination of a refuelling sequence, when the aircraft to be refuelled are more numerous than the refuelling booms at the disposal of the refuelling aircraft.

For its effective refuelling, an aircraft must approach the refuelling aircraft very closely from behind, enter its wake to the point of suffering turbulence, grasp a refuelling boom and remain connected to it for the time required to transfer the required quantity of fuel, while continuing to be subject to the turbulence of the refuelling aircraft, making piloting difficult with the risk of a refuelling operation in progress being prematurely interrupted at any time.

The determination of a refuelling sequence when the aircraft of a fleet need to refuel one after the other at the same refuelling boom causes problems in the order of presentation of the aircraft at the refuelling boom and the number of passes at the refuelling boom of each aircraft in the fleet in order to obtain the desired quantity of fuel. These problems must be resolved in such a way as to optimise, at any time, the in-flight refuelling operation, in other words so that the operational capacity of the fleet is at its maximum if the in-flight refuelling operation has to be prematurely interrupted for whatever reason, and to minimise the number of passes of the aircraft at the refuelling boom since each pass is a difficult and costly operation in terms of manoeuvring.

The optimisation at any time of the in-flight refuelling of a fleet of aircraft, from the same refuelling boom, is normally achieved by organising a queue which is arranged in order of decreasing fuel requirements. In fact, a fleet of aircraft generally comprises aircraft with comparable characteristics, having reserves and fuel consumption of the same order, so that the flying ranges of the aircraft which comprise the fleet are, in most cases, inversely proportional to their fuel requirements.

It is preferable to refuel an aircraft in a single pass at the refuelling boom, but this is not always possible since the flying ranges of the other aircraft in the fleet must be taken into account. In fact, for greater efficiency of the in-flight refuelling of a fleet of aircraft, it is advisable to plan the rendezvous point for the fleet of aircraft with the refuelling aircraft as late as possible, by seeking to get close to, but not begin to use, the safety fuel reserves of the refuelled aircraft, reserves which must enable them, in all circumstances, to reach a diversion landing field under acceptable safety conditions. It may transpire, depending on operational conditions, that the in-flight refuelling rendezvous is later than planned, and that certain aircraft in the fleet can no longer wait the length of time required for refuelling, in a single pass, the preceding aircraft in the refuelling queue. Two solutions are then possible: either to divert the aircraft which can no longer wait to a diversion landing field without proceeding with their in-flight refuelling, in which case the fleet is then disorganised and its mission often compromised, or to shorten the refuelling of the aircraft preceding an aircraft which cannot wait its turn, in order to ensure that it is refuelled in time, before beginning to use its safety reserve. Provided that the aircraft whose refuelling has been shortened have received a sufficient quantity of fuel to allow them to wait, they may complete their refuelling through a new pass at the refuelling boom by rejoining the end of the queue.

In order to refuel a fleet of aircraft from the same refuelling boom of a refuelling aircraft, it is therefore necessary to plan not only the rendezvous point with the refuelling aircraft but also the order of the aircraft which are to be refuelled in the refuelling queue, the number of passes of the aircraft at the refuelling boom,

preferably one, but possibly two, and the quantities of fuel delivered during the different passes. This produces a significant number of options for the in-flight refuelling of a fleet, especially if three or four aircraft are involved.

The organisation of in-flight refuelling of a fleet of aircraft is currently the responsibility of a refuelling commander on board the refuelling aircraft, who decides on the refuelling procedure, the order of the aircraft which are to be refuelled in the refuelling queue, the number of passes at the refuelling boom and the quantities of fuel delivered during each pass, in accordance with the requirements of the fleet commander and his personal experience. It is very difficult to find the optimum solution for the performance of the mission of the fleet in the case of prolonged delays in relation to the safety reserves of one or more aircraft in the fleet. Furthermore, this may result, for the pilot of a refuelled aircraft and also for the commander of the fleet, in the need to perform a supervision and planning operation during a refuelling phase, in addition to the already difficult piloting operation.

It is therefore important to facilitate the organisation of in-flight refuelling of a fleet of aircraft, either on a mission or in convoy.

The object of the present invention is to provide a method for managing the in-flight refuelling of a fleet of aircraft which can be automated, i.e. can be implemented by a computer, also enabling the proposal of an optimum refuelling sequence, if this exists, while minimising the number of passes, i.e. connection operations at the refuelling boom of the refuelling aircraft, not only from the point of view of the fleet as a whole, but also from the point of view of each of the aircraft which makes up the fleet, while ensuring, for the aircraft in the fleet, the provision, at the end of the refuelling operation, of the required fuel quantities, on the basis of the knowledge of the refuelling rendezvous point, the number of aircraft in the fleet, the quantities of fuel required by the aircraft in the fleet at the end of the refuelling, and the distances, in relation to the refuelling rendezvous point, on the route followed by the refuelling aircraft, the limit points which can be reached by the various aircraft in the fleet without refuelling and without drawing on their fuel safety reserves.

The object of the invention is also to provide a management method of the aforementioned type, enabling a refuelling sequence to be devised in such a way that, during its performance, the operational capacity of the fleet is maximised during the refuelling operations.

The object of the invention is to provide a method for managing the in-flight refuelling of a fleet of n aircraft A1,...., An from the same refuelling boom of a refuelling aircraft which enables a refuelling sequence to be devised taking account of a refuelling rendezvous point P, the number n of aircraft in the fleet, the quantities of fuel Q1, ..., Qn required by the aircraft A1,...., An of the fleet at the end of refuelling, and the maximum distances L1,...., Ln which can be covered by each aircraft in the fleet while waiting for the start of refuelling, these maximum distances L1,...., Ln corresponding to the distances between the refuelling rendezvous point P along the route followed by the refuelling aircraft and the limit points which can be reached by the various aircraft in the fleet without refuelling and without drawing on their fuel safety reserves. This method is remarkable in that it comprises the following steps:

- initial consideration of an arbitrary refuelling sequence defined by the queuing of aircraft in the fleet according to an arbitrary order A1,...., An, and a single pass of each aircraft in the fleet at the refuelling boom,
- viability testing of the refuelling sequence concerned, comprising the expression, in distances D1,...., Dn, to be travelled by the refuelling aircraft, the time required in order to supply the aircraft in the fleet with the planned quantities of fuel during their passes at the refuelling boom, and the verification, going down the queue, that each aircraft A1,...., An will begin its refuelling in time, i.e. before the refuelling aircraft has travelled a distance greater than the maximum distance which can be covered Ln,...., L1 by the aircraft concerned,
- if no aircraft is detected which will be too late in starting its refuelling, confirmation of the viability and adoption of the tested refuelling sequence,

- if an aircraft is detected which will be too late in starting its refuelling, modification of the test refuelling sequence to shorten the waiting time of this aircraft, attempting to allow it to refuel in time, the modification of the sequence consisting in dividing into two passes the refuelling of one or more aircraft which precedes the aircraft concerned in the queue, a first shortened pass at the refuelling boom enabling an aircraft to receive a minimum quantity of fuel, sufficiently increasing its flying range so that it can rejoin the end of the queue and wait for a second pass at the refuelling boom without drawing on its safety fuel reserve, the aircraft whose refuelling is divided into two being selected in such a way as to minimise the number of passes at the refuelling boom, the choice initially falling on the aircraft, if it exists, which is upstream in the queue and whose division of refuelling into two passes at the refuelling boom most closely approximates, in terms of a higher value, the desired time gain, then on two aircraft, if they exist, placed upstream in the queue, whose refuelling divisions most closely approximate, in terms of a higher value, the desired time gain, and so on, the absence of a solution in the choice of aircraft with divided refuelling resulting in realisation of the impossibility of refuelling the entire fleet, whereas the existence of a solution results in a modified refuelling sequence proposal, and
- consideration of any modified refuelling sequence proposal and testing of its viability by restarting from the second step of the method.

Advantageously, in the case of a negative viability test on the arbitrary refuelling sequence initially considered, said sequence is replaced by a "nominal" refuelling sequence defined by queuing the aircraft in the fleet in an order corresponding to the decreasing quantities of fuel required and by a single pass of each aircraft in the fleet at the refuelling boom, the first aircraft in the queue being the aircraft which required the largest quantity of fuel.

Advantageously, in the case of a negative viability test on the arbitrary refuelling sequence initially considered, said sequence is replaced by a refuelling sequence

defined by queuing the aircraft in the fleet in an order corresponding to increasing maximum distances which can be covered L1, L2, ..., Ln, the first aircraft in the fleet being the aircraft which can cover the shortest maximum distance.

Advantageously, in the case of a negative viability test on a refuelling sequence due to the identification of an aircraft in the fleet which will be too late in starting its refuelling, the choice of the aircraft located upstream in the queue whose refuelling is divided up results from systematic virtual tests carried out going back along the queue, taking account of one aircraft, then two, including the aircraft achieving the greatest time gain, then three, two of which achieve the greatest time gain, and so on.

Advantageously, the duration of the first shortened pass at the refuelling boom of a divided refuelling operation is fixed at the same value DMIN for all the aircraft in the fleet, said duration being gradually increased, once the refuelling sequence under consideration has passed the viability test, until the flying range limit is reached for one of the aircraft in the fleet.

Advantageously, the choice of the aircraft in the fleet whose refuelling is divided up results from the verification of observance of a succession of inequality relationships between the travelling distances D1,...., Dn required by the refuelling aircraft to deliver the quantities of fuel required during each pass of the aircraft in the fleet at the refuelling boom and the maximum distances which can be covered by each aircraft in the fleet.

Other characteristics and advantages of the invention will be set out in the description below of an embodiment presented by way of example. This description will be provided with reference to the drawing, in which:

- Figure 1 is a diagram showing the operational conditions of a refuelling sequence of a fleet of aircraft on a surveillance mission,

- Figure 2 is a diagram showing the different options for the fleet of aircraft to be refuelled to connect to the refuelling aircraft in terms of observance of the rendezvous point,
- Figure 3 is a diagram illustrating the refuelling operation along the axis (in the case of a convoy flight, for example),
- Figure 4 is a diagram illustrating the problem posed by the flying range limit of an aircraft which is to be refuelled,
- Figure 5 is a table summarising the different refuelling sequences proposed according to the various flying range problems which may arise in a fleet of the two aircraft,
- Figure 6 is a table summarising the different refuelling sequences proposed according to the various flying range problems which may arise in a fleet of three aircraft, and
- Figures 7a and 7b are tables summarising the different refuelling sequences proposed according to the various flying range problems which may arise in a fleet of four aircraft.

During a long-distance surveillance mission requiring one or more in-flight refuelling operations, a fleet 1 of aircraft follows a route 10 above the surveillance zone which was predefined in time and place during the mission preparation. This often looped route is shown in Figure 1 by a "surveillance" hippodrome. The refuelling operations of the fleet 1 of aircraft are performed by a refuelling aircraft 2 circulating in a safety zone as close as possible to the surveillance zone.

During the refuelling operations, the aircraft in the fleet 1 must manoeuvre in order to seize a refuelling boom trailed by the refuelling aircraft 2, and must stay connected to it for the duration of the fuel transfer. Since these manoeuvres are difficult, they are performed while the aircraft are flying in a straight line. To do this, when the refuelling aircraft 2 follows a looped route in order to remain in a safety zone close to the surveillance zone, it often describes a flattened loop 20 presenting two straight-line outgoing and incoming branches, favourable for the refuelling operations. This flattened looped 20 is also shown in the form of a hippodrome.

Since the position in time and space of the fleet 1 of aircraft on the surveillance hippodrome 10 and the position of the refuelling aircraft 2 on the refuelling hippodrome 20 were planned during the mission preparation, the place 11 and the time when the fleet 1 of aircraft leaves the surveillance hippodrome 10 in the direction of the refuelling aircraft 2 must be optimised so that the aircraft in the fleet 1 can remain for the maximum amount of time in the surveillance zone. This optimisation can be carried out either on the basis of a time agreed during the mission preparation or during the mission for the rendezvous with the refuelling aircraft 2, or on the basis of the flying ranges of the aircraft in the fleet 1 which determine the limit point beyond which the fleet 1 must stop its surveillance mission in order to refuel. This involves not only the determination of the place 11 and the time of departure of the fleet 1 of aircraft towards the refuelling aircraft 2, the determination of a place 21 and a time of rendezvous with the refuelling aircraft 2 on the refuelling hippodrome 20, of a meeting trajectory 22, of diversion landing fields and diversion limit points and trajectories for each aircraft in the fleet, taking account of their flying ranges before refuelling.

From a practical point of view, there may be a time deviation in the actual rendezvous, due either to the refuelling aircraft or the fleet of refuelled aircraft, so that the rendezvous may take place early, as indicated by II in Figure 2, or late, as indicated by III in Figure 2, thereby requiring periodic updating of the rendezvous place and time, the meeting trajectory, the diversion landing fields and the diversion limit points and trajectories of the different aircraft in the fleet.

During a refuelling of the fleet 1 of aircraft at a single refuelling boom, the aircraft in the fleet 1 must form a queue in order to grasp, each in turn, the refuelling boom. Figure 3 shows the performance of a refuelling procedure during which the refuelling aircraft 2 and the fleet 1 of aircraft fly in a line, a first aircraft of the fleet grasping the refuelling boom for a duration DEB•1 to FIN•1, then a second aircraft of the fleet for a duration DEB•2 to FIN•2, and so on, the convoy formed by the fleet 1 of aircraft

and the refuelling aircraft 2 passing close to different diversion landing fields: field A, field B, field C. Field D, etc.

In order for an in-flight refuelling sequence of a fleet 1 of aircraft to be viable, i.e. acceptable from the point of view of the safety of the refuelled aircraft, it must respect the fuel-dependent flying range constraints of the aircraft which are to be refuelled in the fleet 1, i.e. all of the aircraft which are to be refuelled start their refuelling operations before drawing on their safety fuel reserves which are intended to keep them at all times within range of a landing field to which they can divert in the event of a problem.

Observance by an in-flight refuelling sequence of the fuel-dependent flying range constraints of the refuelled aircraft can be verified when the place and time of the rendezvous with the refuelling aircraft are known with sufficient precision. In fact, it is then possible:

- to plan the quantities of fuel which each of the aircraft in the fleet 1 will still have when they arrive at the rendezvous point with the refuelling aircraft 2 on the basis of the knowledge of their actual reserves, their consumption and the distance they have still to cover up to the rendezvous point, and to deduce therefrom the maximum lengths of the routes that they are capable of travelling in the company of the refuelling aircraft 2 before drawing on their safety reserves,
- to plan the refuelling waiting times of each aircraft in the fleet from the start of the rendezvous on the basis of the knowledge of the order of the aircraft in the refuelling queue and the duration of their pass at the refuelling boom, expressed taking account of the durations of the connection and disconnection manoeuvres at the refuelling boom and the durations of transfer of the required quantities of fuel,
- to translate the estimated refuelling waiting times into distances travelled, and

- to verify that the refuelling waiting times imposed on the aircraft in the fleet 1, expressed as distances covered, are less than the maximum lengths that they are capable of travelling.

Figure 4 illustrates the problem posed by observance of the fuel-dependent flying range constraints of a refuelled aircraft. The aircraft to be refuelled 3 follows a meeting trajectory 23 with the refuelling aircraft waiting on the refuelling hippodrome 20 for a meeting 24 slightly ahead of the planned rendezvous 21. In its current position, it has an actual fuel reserve which enables estimation, on the basis of the knowledge of its consumption and the trajectory followed, of the quantity of fuel which it will still have on its arrival at the actual rendezvous point 24. Once it has reached the rendezvous point 24, it will fly in convoy with the refuelling aircraft along the refuelling hippodrome until the in-flight refuelling operation is completed for the entire fleet of which it forms part. During the first part of the flight in convoy, where the aircraft to be refuelled is waiting for refuelling to start, its fuel reserves must not drop below the safety volume corresponding to that which it requires in order to reach a diversion landing field, in this case the field 25. There is therefore a maximum distance which the aircraft 3 can cover during its flight in convoy with the refuelling aircraft while waiting for its refuelling. It is possible to estimate the limit point 26, referred to as the loto point, which the aircraft 3 can reach on the refuelling hippodrome 20 without being refuelled and without beginning to use its safety fuel reserves, since its actual fuel reserve is known, along with the trajectory that it will travel, the manner in which it will do this, and the position of the diversion landing field(s) 25. The distance between this loto point 25 on the refuelling hippodrome 20 and the actual rendezvous point 24 with the refuelling aircraft is the maximum distance L which the aircraft 3 can travel in convoy with the refuelling aircraft while waiting for refuelling. The time required to travel this distance corresponds to the maximum waiting time which the aircraft to be refuelled 3 can allow itself.

The viability of a refuelling sequence of a fleet of aircraft is an important characteristic, since its absence causes the break-up of the formation of the fleet of aircraft and often the abandonment of the performance of the mission. It must

therefore be verified before adopting an in-flight refuelling sequence proposal. Moreover, the proposed in-flight refuelling sequence must be optimum, i.e. must minimise the number of passes at the refuelling boom, since the connection manoeuvres are difficult operations, and must maintain the operational capacity of the fleet at its maximum during its performance.

In order to minimise the number of passes at the refuelling boom, an attempt is made to refuel the aircraft in the fleet in a single pass at the refuelling boom, unless waiting is impossible due to the inadequate flying range of one or more aircraft, in which case up to two passes at the refuelling boom are authorised for each aircraft which is to be refuelled.

In order to maintain maximum operational capacity of the fleet of refuelled aircraft, an attempt is made to devise the in-flight refuelling sequence which corresponds as closely as possible to the delivery of decreasing quantities of fuel during successive passes of refuelled aircraft at the refuelling boom, the passes at the refuelling boom becoming more viable as the quantity of transferred fuel increases.

The optimisation of an in-flight refuelling sequence of a fleet of aircraft consists in determining an optimum order of the queue according to which the aircraft must present themselves at the refuelling boom, supposing that each of them makes at most two passes, the second pass taking place after all the first passes, by the aircraft rejoining the queue, then in determining the duration of the passes in such a way as to eliminate as far as possible the second passes, while observing the waiting time constraints imposed by the flying ranges of the aircraft which are to be refuelled.

If n is the number of aircraft in the fleet, 1, 2 ,..., n is the order adopted for the queue at the refuelling boom, which is preferably the order in terms of decreasing quantities of fuel to be transferred since a fleet often comprises aircraft which have very similar consumption levels and the aircraft with the shortest flying range are those requesting the most fuel, D11+D12 is the distance covered by the refuelling aircraft,

the time to transfer in two passes the quantity of fuel required by the first aircraft in the queue, D21+D22 is the distance covered by the refuelling aircraft, the time to transfer in two passes the quantity of fuel required by the second aircraft in the queue, and so on, Dn1+Dn2 being the distance covered by the refuelling aircraft, the time to transfer in two passes the quantity of fuel required by the last aircraft in the queue, L1, L2,...., Ln are the distances between the rendezvous point and the different loto points of the aircraft in the fleet, observance of the fuel constraints of the aircraft in the fleet is translated by the following condition:

$$L(i+1) \geq \sum_{j=1}^{i} Dj1$$

Minimisation of the number of passes at the refuelling boom eliminates second passes as far as possible, while ensuring observance of the preceding condition.

Optimisation of the refuelling sequence imposes the fixing of a minimum duration DMIN for a first pass sufficient for the transfer of a quantity of fuel which sufficiently increases the flying range of the refuelled aircraft so that it can rejoin the end of the queue and wait for its additional refuelling before beginning to use its emergency fuel reserve, followed by the increase in the durations of the first passes up to the maximum levels rendered possible by the waiting capacities of the aircraft placed behind in the queue.

A method for managing the in-flight refuelling of a fleet of aircraft is proposed below which enables the determination of an optimum refuelling sequence taking account of the flying ranges of the aircraft of a fleet, based on the rendezvous point of the fleet of aircraft with the refuelling aircraft, the required fuel quantities, the distances between the rendezvous point and the different loto points of the aircraft which are to be refuelled and a minimum duration for a pass at the refuelling boom.

During a first step, account is taken of an arbitrary refuelling sequence based on queuing of the aircraft in a fleet according to an order A1, A2, ..., An proposed, for

example, by the fleet commander, and on a single pass by each aircraft in the fleet at the refuelling boom, these single passes requiring travelling distances D1, D2, ..., Dn, the viability of this arbitrary refuelling sequence, i.e. observance of the flying ranges of the aircraft in the fleet, being ascertained through verification of all of the following conditions:

$$\sum_{i=1}^{j} Di < L(j+1) \quad \text{where} \quad 1 \le j \le n-1 \qquad (1)$$

If these conditions are observed in their entirety, the arbitrary refuelling sequence is possible and the refuelling management method can stop.

If one of the preceding conditions is not observed, the arbitrary refuelling sequence is rejected as it poses a flying range problem for at least one of the aircraft in the fleet.

In the event of rejection of the arbitrary refuelling sequence, the method continues with the actual determination of an optimum refuelling sequence. If an aircraft is identified which begins its refuelling outside the time permitted by its flying range, this entails modification of the refuelling sequence under test in order to shorten the waiting time of this aircraft so that it can refuel within the time available. This modification consists in dividing into two passes the refuelling operation(s) of one or more aircraft which precede the aircraft concerned in the queue, a shortened first pass at the refuelling boom enabling an aircraft to receive a minimum quantity of fuel, sufficiently increasing its flying range so that it can rejoin the end of the queue and wait for a second pass at the refuelling boom without drawing on its safety fuel reserve.

This/these division(s) must be effective, i.e. they must achieve the desired aim of refuelling, within the time available, of the aircraft which poses a flying range problem, while imposing a minimum number of manoeuvres and maintaining maximum operational capacity of the fleet throughout the refuelling operation. The number of aircraft whose refuelling operations are divided into two must be

minimised, and the quantities of fuel delivered during these first passes must be maximised.

In order to minimise the number of passes at the refuelling boom, the aircraft posing a flying range problem are treated in the order of the queue. For each aircraft identified as posing a flying range problem, it is ascertained whether one or more divisions of the refuelling of aircraft positioned upstream in the queue would allow this problem to be avoided. The choice of division of refuelling first falls on the aircraft, if it exists, which is located in the queue upstream of the aircraft posing a flying range problem and of which the refuelling division into two passes at the refuelling boom most closely approximates, in terms of a higher value, the desired time gain, then, if none permits the anticipated time gain, on two aircraft, if they exist, located in the queue upstream of the aircraft posing a flying range problem, whose refuelling divisions most closely approximate, in terms of a higher value, the desired time gain, and so on, the absence of a solution in the choice of aircraft with divided refuelling resulting in realisation of the impossibility of refuelling the entire fleet, whereas the existence of a solution results in a modified refuelling sequence proposal, the viability of which is tested before being accepted.

If the arbitrary refuelling sequence initially proposed turns out to be unviable, this fact is exploited, before one or more divisions into two refuelling passes of the aircraft in the fleet are envisaged, in order to replace the sequence with the "nominal" refuelling sequence, still based on a single pass by the refuelled aircraft, but with a queuing order corresponding to decreasing quantities of transferred fuel, the aircraft first served being the aircraft with the greatest requirement, the second with a lower requirement and so on through to the last. This nominal refuelling sequence in fact has the best prospects of maintaining the maximum operational capacity of the fleet during the refuelling operation insofar as it comprises aircraft of comparable performance. It corresponds to the distances D1, D2, ..., Dn which are necessary for the refuelling in a single pass of the aircraft in the fleet forming a decreasing sequence:

$$D1 \geq D2 \geq \ldots \geq Dn$$

If it is different from the initial refuelling sequence, the viability of the nominal refuelling sequence is in turn tested. If it turns out to be viable, all the conditions (1) being satisfied, it is adopted and the process of determining the refuelling sequence is stopped. If it turns out to be unviable, one or more aircraft in the fleet having foreseeable flying range problems during the refuelling operation, it is also possible to envisage, through knowledge acquired, a third refuelling sequence, still based on a single pass by the refuelled aircraft, but with a queuing order corresponding to the maximum distances L1, L2,...., Ln which can be covered by the various aircraft in the fleet during the refuelling, forming an increasing sequence:

$$L1 \leq L2 \leq \ldots \leq Ln$$

This third refuelling sequence based on a single pass by refuelled aircraft is envisaged through knowledge acquired, since it normally corresponds to the nominal refuelling sequence. If it is different from the nominal refuelling sequence, its viability is tested. If it turns out to be viable, the process of determining the refuelling sequence is stopped.

If the preceding refuelling sequences based on a single pass by refuelled aircraft all turn out to be unviable, the nominal refuelling sequence is resumed and an attempt is made to render it viable through division into two refuelling passes for one or more aircraft in the fleet.

Before envisaging refuelling divisions, there is a need to verify that they will be able to resolve the flying range problem(s) which arise when testing if the aircraft to be refuelled, placed in the queue in the adopted order, can wait to be refuelled until the end of the first passes of the preceding aircraft, assuming that these first passes last a minimum time corresponding to a minimum distance travelled DMIN for each first pass at the refuelling boom, which will verify fulfilment of the following conditions:

$$i \times DMIN \leq L(i+1) \quad where\ 1 \leq i \leq n-1, \qquad (2)$$

the minimum distance DMIN being chosen according to operational conditions in such a way as to enable a transfer of a minimum quantity of fuel which, at the same time, renders viable a pass at the refuelling boom, allows the refuelled aircraft to

rejoin the end of the queue and wait for a second pass at the refuelling boom without encountering any flying range problem.

If the conditions (2) are not fulfilled, the problem is considered to be irresolvable.

If the conditions (2) are satisfied, operations are carried out according to the number of aircraft which make up the fleet which is to be refuelled.

Case involving a fleet of two aircraft

The aircraft posing a flying range problem during refuelling is bound to be the second aircraft. The solution therefore consists in dividing the refuelling of the first aircraft into two passes corresponding to the distances of travel D11, D12 while refuelling the second aircraft in a single pass corresponding to a distance of travel D2 interspersed between the distances of travel D11 and D12 required for refuelling the first aircraft.

In the refuelling, the distance travelled in formation by the refuelling aircraft and the fleet is made up of the succession D11, D2, D12. To maintain the fleet at its maximum operational capacity, the first pass at the refuelling boom of the first aircraft of the fleet is extended to the maximum permitted by the flying range of the second aircraft in the fleet. Instead of being reduced to the minimum distance of travel DMIN, its distance of travel is prolonged in order to reach the maximum distance of travel L2 authorised by the flying range of the second aircraft.

The adopted refuelling sequence is therefore defined by the following distance of travel:

$$D11, D2, D12$$

where:

$$D11=L2 \text{ and } D12=D1-D11$$

not taking account in the second term of the additional distances required by the second connection manoeuvre at the refuelling boom.

Figure 5 is a diagram illustrating the different cases which may arise during in-flight refuelling, on the same refuelling boom, of a fleet of two aircraft, for a given refuelling order 1, 2, quantities of fuel required by each of the two aircraft in the fleet and the minimum quantity of fuel which can transferred during a pass at the refuelling boom, estimated as the distance of travel D1, D2, DMIN required by the refuelling aircraft for their transfers, and the flying range of the second aircraft in the fleet estimated as the maximum coverable distance L2.

The diagram in Figure 5 shows an axis 30 graduated in distance with, as a variable, the value of the maximum distance L2 which can be covered by the second aircraft in the fleet during the refuelling operation, counting from an origin O coinciding with the actual rendezvous point of the fleet with the refuelling aircraft. The minimum distance DMIN travelled by the refuelling aircraft during a first refuelling pass comprising two [...?], and the distance D1 which the refuelling aircraft must cover in order to transfer to the first aircraft the entire quantity of fuel which it requests in a single pass, are plotted on this axis 30 graduated in distance.

If the maximum distance L2 which the second aircraft can travel while waiting to be refuelled is greater than the distance required by the refuelling aircraft to entirely refuel the first aircraft, there is no need to divide up the refuelling of the first aircraft. The situation shown in square 31 arises. In this situation, the adopted refuelling sequence is that of D1, D2, with successive refuelling of the two aircraft in a single pass at the refuelling boom.

If the maximum distance L2 which the second aircraft can travel while waiting to be refuelled is less than DMIN, the refuelling sequence is impossible without compromising the flying range of the second aircraft. It must be abandoned, or the refuelling order of the two aircraft in the fleet must at least be reversed. This case corresponds to square 32, indicated by 0.

If the maximum distance L2 which the second aircraft can travel is between DMIN and D1, observance of the flying range of the second aircraft imposes a division into two passes of the refuelling of the first aircraft, with a first pass D11 at the refuelling boom, greater than DMIN but less than L2, and an additional second pass at the refuelling boom D12 following the refuelling D2 of the second aircraft. The situation corresponds to square 33. In this situation, the adopted refuelling sequence is D11, D2, D12, i.e. a refuelling sequence comprising a first waiting pass D11 of the first aircraft at the refuelling boom, followed by a single pass D2 of the second aircraft at the refuelling boom, during which the latter receives the entire required quantity of fuel, and the second additional pass D12 of the first aircraft, during which the latter receives the additional fuel to which it was not entitled during its first pass.

Case involving a fleet of three aircraft

As indicated above, two verifications have already been carried out, i.e. for the existence of a flying range problem, revealed by the fact that at least one of the conditions D1<L2 and D1+D2<L3 has not been satisfied, and for the possibility of resolving this flying range problem, revealed by the fact that the conditions DMIN<L2 and 2DMIN<L3 are satisfied.

The flying range problem may concern either the second aircraft or the third aircraft, or both the second and third aircraft of the fleet. This must therefore be precisely identified.

A start is made by checking whether the flying range problem concerns only the second aircraft in the fleet, by once more observing whether the conditions D1<L2 and D1+D2<L3 are satisfied.

The condition D1<L2 may be the only condition not to be satisfied. The flying range problem concerns only the aircraft located in second position in the queue. The condition DMIN<L2 having been checked by assumption, a division into two passes

of the refuelling of the first aircraft enables resolution of the flying range problem of the second aircraft. The refuelling of the first aircraft is then divided up into two passes:

$$D1=D11+D12$$

where:

$$D11=DMIN$$

If the condition DMIN<L2 was not satisfied, there would be no viable solution and no refuelling sequence would be proposed.

The condition D1+D2<L3 may be the only condition not to be satisfied. The flying range problem concerns only the aircraft located in third position in the queue. The condition 2DMIN<L3 having been checked by assumption, this flying range problem can be resolved by one or more divisions into two passes of the refuelling of the aircraft located upstream in the queue. An attempt is first made to establish whether a division into two passes of the refuelling of the second aircraft would suffice to resolve the flying range problem of the third aircraft by checking whether the condition D1+DMIN<L3 is satisfied.

If the condition D1+DMIN<L3 is satisfied, a division into two passes of the refuelling of the second aircraft enables resolution of the flying range problem of the third aircraft. The refuelling of the second aircraft is then divided up into two passes:

$$D2=D21+D22$$

where

$$D21=DMIN$$

If the condition D1+DMIN<L3 is not satisfied, a division into two passes of the refuelling of the second aircraft will not suffice to resolve the flying range problem of the third aircraft. An attempt is then made to establish whether a division into two passes of the refuelling of the first aircraft would resolve this flying range problem by verifying whether the condition DMIN+D2<L3 is satisfied.

If the condition $DMIN+D2<L3$ is satisfied, a division into two passes of the refuelling of the first aircraft enables resolution of the flying range problem of the third aircraft. The refuelling of the first aircraft is then divided up into two passes:

$$D1=D11+D12$$

where

$$D11=DMIN$$

If the condition $DMIN+D2<L3$ is not satisfied, a division into two passes of the refuelling of the second or first aircraft will not enable resolution of the flying range problem of the third aircraft. The refuelling of the first and second aircraft must then be divided into two passes:

$$D1=D11+D12$$
$$D2=D21+D22$$

where

$$D11=DMIN$$
$$D21=DMIN$$

If the condition $2DMIN<L3$ was not satisfied, there would be no viable solution and no refuelling sequence would be proposed.

The conditions $D1<L2$ and $D1+D2<L3$ may not be satisfied simultaneously, showing that the flying range problems simultaneously affect the two aircraft located in second and third positions in the refuelling queue. Since the conditions $DMIN<L2$ and $2DMIN<L3$ are assumed to be satisfied, the divisions into two passes of the refuelling of the first and second aircraft enable resolution of the flying range problems of the second and third aircraft in the fleet:

$$D1=D11+D12$$

$$D2=D21+D22$$

where

$$D11=DMIN$$

$$D21=DMIN$$

When a division solution enabling resolution of the flying range problems has been found, it is optimised, from the point of view of the operational capacity of the fleet during refuelling, by extending the durations of the first pass(es) at the refuelling boom up to the maximum permitted by observance of the flying ranges of the aircraft in the fleet.

In the case of the single division of refuelling of the second aircraft in the refuelling queue formed by the fleet, the first pass D21 of the second aircraft in the fleet at the refuelling boom is extended from DMIN to L3-D1. The proposed refuelling sequence is then:

$$D1,\ D21=L3-D1,\ D3,\ D22=D2-D21$$

In the case of the single division of refuelling of the first aircraft in the refuelling queue formed by the fleet, the first pass D11 of the first aircraft in the fleet at the refuelling boom is extended from DMIN to min(L2, L3-D2). The proposed refuelling sequence then becomes:

$$D11=\min(L2,\ L3-D2),\ D2,\ D3,\ D12=D1-D11$$

In the case of the divisions of the refuelling of the first and second aircraft in the queue formed by the fleet, the first passes D11 and D21 of the first and second aircraft at the refuelling boom are extended by successive increments from the value DMIN to a limit value still enabling observance of the conditions $D11<L2$ and $D11+D21<L3$. The proposed refuelling sequence is then becomes:

$$D11=DMIN+K,\ D21=DMIN+K,\ D3,\ D12=D1-D11,\ D22=D2-D21$$

where K is the maximum increase in the distance DMIN enabling observance of the conditions:

$$D11<L2 \text{ and } D11+D21<L3$$

Figure 6 is a table summarising, in the case of refuelling of a fleet of three aircraft, all the solutions and absences of a solution entailing refuelling division which render a refuelling sequence viable with a minimum number of passes at the refuelling boom. This table has two dimensions, one having as its variable the flying range L2 of the second aircraft in the refuelling queue, and the other the flying range L3 of the third aircraft in the refuelling queue. It is plotted according to the values of the flying ranges L2 and L3 of the aircraft in the fleet located in second and third positions in the refuelling queue, compared with the values of the quantities of fuel D1, D2, D3 required by the three aircraft in the fleet and assumed to be decreasing values, and the minimum quantity DMIN of fuel which can be transferred during a pass at the refuelling boom, the different variables being expressed as the distances travelled by the refuelling aircraft.

The table in Figure 6 shows that the same division solution may be appropriate for different values of the flying ranges of the aircraft in the fleet located in second and third positions in the queue.

Only two cases are dealt with by a specific solution.

The first case referenced as 40 corresponds to the absence of any refuelling division, the aircraft located in second position in the refuelling queue being able to wait until the end of the refuelling of the first aircraft, the condition $D1<L2$ being satisfied, and the aircraft placed at the end of the queue being able to wait for the refuelling of the first two aircraft, the condition $D1+D2<L3$ being satisfied.

The second case referenced as 41 corresponds to the division of the refuelling of only the aircraft located in second position in the queue, this aircraft having no flying range problem, the condition $D1<L2$ being satisfied, whereas the aircraft located in

third position in the queue has a problem, its flying range L3 being within the range $D1+DMIN<L3<D1+D2$.

The other cases can be divided into three categories referenced by 42, 43, 44.

Category 42 corresponds to the division of the refuelling of only the first aircraft in the queue, which is justified, either in the case of a flying range problem for only the aircraft located in second position in the queue, whereby the flying range L2 of this second aircraft lies within the range $DMIN<L2<D1$, or, in the case of a flying range problem for only the aircraft located in third position in the queue, whereby the flying range L3 of this third aircraft lies within the range $D2+DMIN<L3<D1+DMIN$, or again, in the case of flying range problems of the aircraft located in second and third positions in the queue, whereby the flying ranges L2 and L3 lie within the ranges $DMIN<L2<D1$ and $D2+DMIN<L3<D1+D2$.

Category 43 corresponds to the double division of the refuelling of the aircraft located in first and second positions in the queue, when the aircraft placed in third position in the queue has a flying range problem such that its flying range L3 is located within the range $2DMIN<L3<D2+DMIN$, whereby the aircraft located in second position in the queue may or may not have a flying range problem.

Category 44 corresponds to all irresolvable cases, where at least one of the aircraft located in second or third position in the refuelling queue cannot wait the minimum period DMIN of a first refuelling pass of the first aircraft.

The grouping into categories of the division solutions which are shown in the table in Figure 6 enables an accelerated search method for the division or non-division solution for the refuelling of a fleet of three aircraft, enabling resolution of any flying range problems with a minimum number of passes at the refuelling boom. This accelerated search method is derived from the general method described above, and makes use of a technique for selecting the most suitable solution through successive eliminations.

This accelerated search begins by testing the first individual case 40 where no division is necessary. This case 40, involving the absence of refuelling divisions already envisaged at the beginning of the general method described above, is adopted and the search is stopped if the aircraft located in second and third positions in the refuelling queue can wait their turn, i.e. if the conditions $D1<L2$, $D1+D2<L3$ are satisfied. Otherwise, it is rejected and the search is resumed.

When the case 40 involving the absence of refuelling divisions is rejected due to flying range problems, whereby at least one of the conditions $D1<L2$ or $D1+D2<L3$ is not satisfied, a check is carried out to ascertain whether the flying range problem(s) can be resolved through division of refuelling into two passes. This check is carried out by observing compliance with the conditions $DMIN<L2$ and $2DMIN<L3$.

If one of the conditions $DMIN<L2$ or $2DMIN<L3$ is not satisfied, the refuelling divisions will not enable resolution of the flying range problems. The situations of the category 44 arise and refuelling must be designated as impossible.

If the two conditions $DMIN<L2$ and $2DMIN<L3$ are satisfied, the search is resumed through observation of the individual second case 41 corresponding to the division of the refuelling of only the aircraft located in second position in the queue. This second individual case 41 is adopted and the search is stopped in the case of simultaneous observance of the conditions $D1<L2$ and $D1+DMIN<L3$. If not, it is in turn rejected and the search is resumed.

If the two individual cases 40 and 41 have been eliminated, category 42 corresponding to the different cases involving inadequate flying range and requiring the division of the refuelling of only the aircraft located in first position in the queue is envisaged. Given the elimination of the two individual cases 40 and 41, the only condition to be satisfied for the selection of this category 42 is the condition $D2+DMIN<L3$. If this condition is satisfied, the category 42 corresponding to the division of the refuelling of only the second aircraft in the queue is adopted and the

search is stopped. If not, the category 42 is rejected in turn and is the search resumed.

If the two individual cases 40 and 41, and also the category 42 have been eliminated, category 43 corresponding to the different cases of lack of flying range which can be resolved by the divisions of the refuelling of the first and second aircraft in the queue, which is the only remaining category, is adopted and the search is stopped.

As above, when one or two refuelling divisions have been adopted (individual case 41 and categories 42 and 43), the durations of the first passes at the refuelling boom are maximised, as a result of which, for the individual case 41 of a division of the refuelling of only the aircraft located in second position in the queue, the first pass D21 is extended from DMIN to L3–D1, for category 42 corresponding to a division of the refuelling of only the aircraft located in first position in the queue, the first pass D11 is extended from DMIN to min(L2, L3-D2) and, for category 43 corresponding to the divisions of the refuelling of the aircraft located in first and second positions in the queue, the durations of the first passes D11 and D21 are extended from DMIN to the maximum value which enables observance of the flying range conditions, i.e. $D11<L2$ and $D11+D21<L3$.

Case involving a fleet of four aircraft

The general method for searching for a refuelling sequence which observes the flying ranges of the aircraft in the fleet and minimises the passes at the refuelling boom, as described above, the application of which to a fleet of three aircraft has just been described in detail, can easily be extended to a fleet of four aircraft. This results in a number of solutions and absences of solutions which can be summarised in a three-dimensional table, with a first dimension according to the flying range L2 of the second aircraft in the refuelling queue, a second dimension according to the flying range L3 of the third aircraft in the refuelling queue and a third dimension according to the flying range L4 of the fourth aircraft in the refuelling queue. This three-

dimensional table can be shown as a two-dimensional representation by projecting it along one of its axes. It is shown in Figures 7a and 7b which result from a projection in relation to the axis of its first dimension L2 and which deal with the case in which the aircraft to be refuelled are placed in a refuelling queue order corresponding to quantities of fuel required in descending order.

More precisely the two-dimensional table in Figure 7a summarises the solutions suitable for situations in which the aircraft located in second position in the refuelling queue has no flying range problem, the condition D1<L2 being satisfied, whereas the two-dimensional table shown in Figure 7b summarises the solutions suitable for situations in which the aircraft located in second position in the refuelling queue has a flying range problem which can be resolved through division of the refuelling of the first aircraft in the queue, the conditions DMIN<L2<D1 being satisfied. Strictly speaking, other situations may arise, but they all correspond to the scenario in which the second aircraft in the refuelling queue has a flying range problem such that it cannot wait until the end of a first pass at the refuelling boom of the first aircraft in the queue. There are then no solutions, resulting in a two-dimensional table which comprises only the category of irresolvable cases, the representation of which provides no specific information.

As shown above in the table in Figure 6, the tables in the Figures 7a and 7b show that the same division solution may be suitable for different values of the flying ranges of the aircraft in the fleet located in second, third and fourth positions in the queue.

Only two cases are examined in a specific solution.

The first case referenced as 50 in Figure 7a corresponds to the absence of any refuelling division, the aircraft located in second position in the refuelling queue being able to wait until the end of the refuelling of the first aircraft, the condition D1<L2 being satisfied for the entire table in Figure 7a, where the aircraft located in third position in the queue can wait until the end of refuelling of the first and second

aircraft, the condition $D1+D2<L3$ being satisfied, and the aircraft located at the end of the queue can wait for the refuelling of the first three aircraft, the condition $D1+D2+D3<L4$ being satisfied.

The second case referenced as 51 in Figure 7a corresponds to the division of the refuelling of only the aircraft located in third position in the queue, whereby neither this aircraft nor those preceding it in the queue have a flying range problem, the condition $D1<L2$ being satisfied for the entire table in Figure 7a, and the condition $D1+D2<L3$ also being satisfied in this case, whereas the aircraft located in fourth position in the queue has a flying range problem, its flying range L4 lying within the range $D1+D2+DMIN<L3<D1+D2+D3$.

The other cases divide up into different categories referenced as 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, shown either in Figure 7a or in Figure 7b.

Starting with Figure 7a, which corresponds to the refuelling situations in which the aircraft in second position in the queue never has a refuelling problem, the condition $D1<L2$ always being satisfied, the following can be defined:

- a category 52 corresponding to the division of the refuelling of only the second aircraft in the queue, justified either in the case of a flying range problem for only the aircraft located in fourth position in the queue such that the flying range L4 of this fourth aircraft lies within the range $D3+D1+DMIN<L4<D2+D1+DMIN$, or, in the case of a flying range problem for only the aircraft located in third position in the queue, such that the flying range L3 of this third aircraft lies within the range $D1+DMIN<L3<D1+D2$, or, in the case of flying range problems of the aircraft located in third and fourth positions in the queue, such that their flying ranges L3 and L4 are located within the ranges $D1+DMIN<L3<D1+D2$ and $D3+D1+DMIN<L4<D1+D2+D3$,

- a category 53 corresponding to the division of the refuelling of only the first aircraft in the queue, justified either in the case of a flying range problem for only the aircraft located in fourth position in the queue such that the flying

range L4 of this fourth aircraft lies within the range $D3+D2+DMIN<L4<D3+D1+DMIN$, or, in the case of a flying range problem for only the aircraft located in third position in the queue, such that the flying range L3 of this third aircraft lies within the range $D2+DMIN<L3<D1+DMIN$, or, in the case of flying range problems of the aircraft located in third and fourth positions in the queue, such that their flying ranges L3 and L4 are located within the ranges $D2+DMIN<L3<D1+D2$ and $D3+D2+DMIN<L4<D3+D1+DMIN$, or within the ranges $D2+DMIN<L3<D1+DMIN$ and $D3+D2+DMIN<L4<D1+D2+D3$.

- a category 54 corresponding to the double division of the refuelling of the aircraft located in second and third positions in the queue, when the aircraft placed in fourth position in the queue has a flying range problem such that its flying range L4 is located within the range $D1+2DMIN<L4<D3+D2+DMIN$, whereby the aircraft located in third position has a flying range L3 which remains greater than D1+DMIN and the aircraft located in second position in the queue has no flying range problem,

- a category 55 corresponding to the double division of the refuelling of the aircraft located in first and third positions in the queue, either in the case of a flying range problem for only the aircraft located in fourth position in the queue such that its flying range L4 lies within the range $D2+2DMIN<L4<D1+2DMIN$, or in the case of flying range problems of the aircraft located in third and fourth positions in the queue such that their flying ranges L3 and L4 lie within the ranges $D2+2DMIN<L4<D1+2DMIN$ and $D2+2DMIN<L3<D1+D2$ or within the ranges $D2+2DMIN<L4<D3+D2+DMIN$ and $D2+DMIN<L3<D1+DMIN$,

- a category 56 corresponding to divisions of the refuelling of the first and second aircraft in the queue, justified either in the case of a refuelling problem for only the aircraft located in fourth position in the queue such that the flying range L4 of this fourth aircraft lies within the range

$D3+2DMIN<L4<D2+2DMIN$, or, in the case of a flying range problem for only the aircraft located in third position in the queue such that the flying range L3 of this third aircraft lies within the range $2DMIN<L3<D2+DMIN$, or, in the case of flying range problems of the aircraft located in third and fourth positions in the queue such that their flying ranges L3 and L4 lie within the ranges $2DMIN<L3<D1+D2$ and $D3+2DMIN<L4<D2+2DMIN$ or in the ranges $2DMIN<L3<D2+DMIN$ and $D3+2DMIN<L4<D1+D2+D3$,

- a category 57 corresponding to divisions of the refuelling of the first, second and third aircraft in the queue if the aircraft located in fourth position in the refuelling queue has a flying range problem such that its flying range L4 lies within the range $3DMIN<L4<D3+2DMIN$, and

- a category 58 corresponding to irresolvable cases, whereby at least one of the aircraft located in third or fourth position in the refuelling queue cannot wait for the duration of the first refuelling passes of the aircraft which precede it in the queue.

Continuing with Figure 7b which corresponds to the refuelling situations in which the aircraft in second position in the queue has a flying range problem which can be resolved through division of the refuelling of the first aircraft in the queue, the following can be defined:

- a category 60 corresponding to the division of the refuelling of only the aircraft in first position in the queue, either in the case of a flying range problem for only the aircraft located in second position in the queue, which, for all the situations shown in the table in Figure 7b, has a flying range L2 within the range $DMIN<L2<D1$, or, in the case of a double flying range problem, that of the aircraft located in second position in the queue, whose flying range L2 lies within the range $DMIN<L2<D1$, and that of the aircraft located in third position in the queue if its flying range L3 lies within the range $D2+DMIN<L3<D1+D2$, or that of the aircraft located in second

position in the queue, whose flying range L2 lies within the range $DMIN<L2<D1$, and that of the aircraft located in fourth position in the queue if its flying range L4 lies within the range $D3+D2+DMIN<L4<D1+D2+D3$, or, in the case of a triple flying range problem, that of the aircraft located in second position in the queue, whose flying range L2 lies within the range $DMIN<L2<D1$, that of the aircraft located in third position in the queue if its flying range L3 lies within the range $D2+DMIN<L3<D1+D2$, and that of the aircraft located in fourth position in the queue, if its flying range L4 lies within the range $D3+D2+DMIN<L4<D1+D2+D3$,

- a category 61 corresponding to divisions of the refuelling of the aircraft located in first and third positions in the queue, in the case of a double or triple flying range problem affecting the aircraft located in second position in the queue whose flying range L2 lies within the range $DMIN<L2<D1$, the aircraft located in fourth position in the queue, if its flying range L4 lies within the range $D2+2DMIN<L4<D3+D2+DMIN$, and possibly the aircraft located in third position in the queue if its flying range L3 remains greater than the D2+ DMIN,

- a category 62 corresponding to divisions of the refuelling of the aircraft located in first and second positions in the queue either in the case of a double flying range problem, that of the aircraft located in second position in the queue whose flying range L2 lies within the range $DMIN<L2<D1$, and that of the aircraft located in third position in the queue, if its flying range L3 lies within the range $2DMIN<L3<D2+DMIN$, or that of the aircraft located in fourth position in the queue, if its flying range L4 lies within the range $D3+2DMIN<L4<D2+2DMIN$, or, in the case of a triple flying range problem, that of the aircraft located in second position in the queue whose flying range L2 lies within the range $DMIN<L2<D1$, and those of the aircraft located in third and fourth positions in the queue whose flying ranges L3 and L4 lie within the ranges $2DMIN<L3<D1+D2$ and $D3+2DMIN<L4<D2+2DMIN$ or $2DMIN<L3<D2+DMIN$ and $D3+2DMIN<L4<D1+D2+D3$,

- a category 63 corresponding to divisions of the refuelling of the aircraft located in first, second and third positions in the queue, either, in the case of a double flying range problem, that of the aircraft located in second position in the queue whose flying range L2 lies within the range DMIN<L2<D1, and that of the aircraft located in fourth position in the queue whose flying range lies within the range 3DMIN<L4<D3+2DMIN, the aircraft located in third position in the queue having no flying range problem, since its flying range L3 satisfies the condition D1+D2<L3, or, in the case of a triple flying range problem, that of the aircraft located in second position in the queue whose flying range L3 lies within the range DMIN<L2<D1, that of the aircraft located in third position in the queue whose flying range L3 lies within the range 2DMIN<L3<D1+D2, and that of the aircraft located in fourth position in the queue whose flying range L4 lies within the range 3DMIN<L4<D3+2DMIN, and

- a category 64 corresponding to irresolvable cases, whereby at least one of the aircraft located in third or fourth position in the refuelling queue cannot wait for the duration of the first refuelling passes of the aircraft which precede it in the queue.

As above, in the case of a fleet comprising three aircraft, grouping into categories of the division solutions shown in the tables in Figures 7a and 7b makes it possible to devise an accelerated search method for the refuelling division or non-division solution for a fleet of four aircraft, enabling the resolution of any flying range problems with a minimum number of passes at the refuelling boom. The accelerated search method is derived from the general method described above and makes use of a technique of successive eliminations in order to select the most suitable solution.

This accelerated search begins by testing the viability of the specific case 50 involving the absence of refuelling divisions, envisaged at the beginning of the

general method described above. The absence of refuelling divisions is adopted and the search is stopped if the aircraft located in second, third and fourth positions in the refuelling queue can wait their turn, i.e. if the conditions D1<L2, D1+D2<L3 and D1+D2+D3<L4 are satisfied. Otherwise the specific case 50 is rejected and the search is continued.

If the case 50 involving absence of refuelling division is not suitable, i.e. if one or more flying range problems arise, an investigation is carried out to ascertain whether these flying range problems can be resolved by refuelling divisions of the aircraft placed upstream in the queue, envisaging categories 58 and 64 which define the impossible situations. Membership of these impossible categories 58 and 64 is indicated by the failure to satisfy one of the conditions DMIN<L2 or 2DMIN<L3 or 3DMIN<L4. If one of these conditions is not satisfied, the search is stopped as it is destined to fail.

If these conditions are satisfied, the flying range problem(s) which resulted in the elimination of the specific case 50 can be resolved by one or more refuelling divisions. The search then continues to find the refuelling division(s) which will result in a minimum number of passes at the refuelling boom, while avoiding the flying range problems.

The case 51 of division of the refuelling of only the aircraft located in third position in the queue is then envisaged. It is adopted and the search is stopped if the conditions D1<L2, D1+D2<L3 and D1+D2+DMIN<L4 are satisfied. If not, it is in turn rejected and the search is continued.

If the two specific cases 50 and 51 have been eliminated, category 52 is envisaged, corresponding to the different cases in which flying range problems occur which can be resolved through division of the refuelling of only the aircraft located in second position in the queue. Taking into account the prior eliminations of the two specific cases 50 and 51, the only conditions to be satisfied for the selection of this category 52 are the conditions D1<L2, D1+DMIN<L3 and D1+D3+DMIN<L4. If these

conditions are all satisfied, the division of the refuelling of only the second aircraft in the queue is adopted and the search is stopped. If not, category 52 is in turn rejected and the search is continued.

If category 52 has been eliminated, categories 53 and 60 are envisaged, corresponding to the different cases in which flying range problems occur which can be resolved by the division of the refuelling of only the aircraft located at the front of the queue. Taking into account the prior eliminations of the two specific cases 50 and 51, and category 52, the only conditions to be satisfied for the selection of these categories are the conditions D2+DMIN<L3 and D3+D2+DMIN<L4. If these conditions are satisfied, the division of the refuelling of only the aircraft located in first position in the queue is adopted and the search is stopped. If not, categories 53 and 60 are rejected and the search is continued.

If the categories 53 and 60 have been eliminated, category 54 is envisaged, corresponding to the different cases in which flying range problems occur which can be resolved through division of the refuelling of only the aircraft located in second and third positions in the queue. Taking into account the prior eliminations of the two specific cases and category 52, the only conditions to be satisfied for the selection of this category are the conditions D1<L2, D1+DMIN<L3 and D1+2DMIN<L4. If these conditions are all satisfied, the divisions of the refuelling of only the aircraft located in second and third positions in the queue are adopted and the search is stopped. If not, category 54 is in turn rejected and the search is continued.

If category 54 has been eliminated, categories 55 and 61 are envisaged, corresponding to the different cases in which flying range problems occur which can be resolved by the division of the refuelling of only the aircraft located in first and third positions in the queue. Taking into account the prior eliminations of the two specific cases 50 and 51, categories 52, 53, 54, 60, the only conditions to be satisfied for the selection of these categories are the conditions D2+DMIN<L3 and D2+2DMIN<L4. If these conditions are satisfied, the division of the refuelling of only the aircraft located in first and third positions in the queue is adopted and the search

is stopped. If not, categories 55 and 61 are in turn rejected and the search is continued.

If the categories 55 and 61 have been eliminated, categories 56 and 62 are envisaged, corresponding to the different cases in which flying range problems occur which can be resolved through division of the refuelling of only the aircraft located in first and second positions in the queue. Taking into account the prior eliminations of the specific cases 50, 51 and categories 52, 53, 54, 55, 60, 61, the only condition to be satisfied for the selection of these categories is the condition D3+2DMIN<L4. If this condition is satisfied, the divisions of the refuelling of only the aircraft located in first and second positions in the queue are adopted and the search is stopped. If not, categories 56 and 62 are in turn rejected and the search is continued.

If categories 56 and 62 have been eliminated as unsuitable, this leaves only categories 57 and 63 corresponding to divisions of the refuelling of the aircraft located in first, second and third positions in the queue. These divisions are adopted, since it was established at the beginning of the search that the flying range problems could be resolved and that the divisions of the refuelling of all the aircraft other then the aircraft located at the front of the queue constitute the ultimate solution.

As above, if one or more refuelling divisions has been adopted, the durations of the first passes at the refuelling boom are maximised, while ensuring that the flying ranges of the aircraft are observed.

Thus, in the cases of division of the refuelling of only the aircraft located in third position in the queue, the first pass D31 at the refuelling boom of the aircraft in third position in the queue is extended from DMIN to L4-D1-D2. In the case of the division of refuelling of only the aircraft located in second position in the queue, the first pass D21 at the refuelling boom of the aircraft in second position in the queue is extended from DMIN to min(L3-D1, L4-D1-D2). In the case of the division of refuelling of only the aircraft located in first position in the queue, the first pass D11 at the refuelling

boom of the aircraft in first position in the queue is extended from DMIN to min(L2, L3-D2, L4-D2-D3).

Similarly, in the case of divisions of the refuelling of the two aircraft located in third and second positions in the queue, the first passes D31 and D21 at the refuelling boom of these two aircraft are extended from DMIN to a value DMAX such that the conditions D1+2DMAX<L4 and D1+DMAX<L3 are satisfied. In the case of divisions of the refuelling of the two aircraft located in third and first positions in the queue, the first passes D31 and D11 at the refuelling boom of these two aircraft are extended from DMIN to a value DMAX such that the conditions D2+2DMAX<L4, DMAX+D2<L3 and DMAX<L2 are satisfied. In the case of divisions of the refuelling of the two aircraft located in second and first positions in the queue, the first passes D21 and D11 at the refuelling boom of these two aircraft are extended from DMIN to a value DMAX such that the conditions D3+2DMAX<L4, 2DMAX<L3 and DMAX<L2 are satisfied.

Similarly, in the case of divisions of the refuelling of three aircraft located in first, second and third positions in the queue, their first passes D31, D21 and D11 at the refuelling boom are extended from DMIN to a value DMAX such that the conditions 3DMAX<L4, 2DMAX<L3 and DMAX<L2 are satisfied.

If the refuelling operations of several aircraft are divided, the search for the common maximum value for their first passes DMAX can be carried out through iterations, starting with the value DMIN and repetitively adding to it an increment INC until one of the conditions imposed by the flying ranges of the aircraft is not satisfied.

The method for in-flight refuelling which has just been described can easily be extended to the refuelling of fleets composed of more than four aircraft, whereby the increase in the number of aircraft in the fleet only serves to increase the options for refuelling divisions.

Once the order of refuelling of the aircraft in the fleet has been established and the aircraft whose refuelling must be divided have been identified, the refuelling times and distances for the quantities of fuel required can be precisely evaluated. It is then possible to estimate the quantities of fuel which the aircraft in the fleet will actually have at the end of the refuelling operation, and to adjust the quantities of fuel transferred to the aircraft so that these estimated quantities correspond to the quantities required. This adjustment, which has consequences for the refuelling times and distances, can be carried out in an iterative manner until acceptable differences are attained between the requirements and estimates of the quantities of fuel provided.

CLAIMS

1. Method for managing the in-flight refuelling of a fleet (1) of n aircraft A1,...., An from the same refuelling boom of a refuelling aircraft (2) which enables a refuelling sequence to be devised taking account of a refuelling rendezvous point P, the number n of aircraft in the fleet (1), the quantities of fuel Q1, ..., Qn required by the aircraft A1,...., An of the fleet (1), and the maximum distances L1,...., Ln which can be covered by each aircraft in the fleet (1) while waiting for the start of refuelling, these maximum distances L1,...., Ln corresponding to the distances between the refuelling rendezvous point P along the route followed by the refuelling aircraft (2) and the limit points which can be reached by the various aircraft in the fleet (1) without refuelling and without drawing on their fuel safety reserves, said method being characterised by the following steps:

- initial consideration of an arbitrary refuelling sequence defined by the queuing of aircraft in the fleet (1) according to an arbitrary order A1,...., An, and a single pass of each aircraft in the fleet (1) at the refuelling boom,
- viability testing of the refuelling sequence concerned, comprising the expression, in distances D1,...., Dn, to be travelled by the refuelling aircraft (2), the time required in order to supply the aircraft in the fleet (1) with the planned quantities of fuel during their programmed passes at the refuelling boom, and the verification, going down the queue, that each aircraft A1,...., An will begin its refuelling in time, i.e. before the refuelling aircraft (2) has travelled a distance greater than the maximum distance which can be covered L2,...., Ln [*sic*] by the aircraft concerned,
- if no aircraft is detected which will be too late in starting its refuelling, confirmation of the viability and adoption of the tested refuelling sequence,
- if an aircraft is detected which will be too late in starting its refuelling, modification of the test refuelling sequence to shorten the waiting time of this aircraft, attempting to allow it to refuel in time, the modification of the sequence consisting in dividing into two passes the refuelling of one or more aircraft which precedes the aircraft concerned in the queue, a first shortened pass at the refuelling boom enabling an aircraft to receive a minimum

quantity of fuel, sufficiently increasing its flying range so that it can rejoin the end of the queue and wait for a second pass at the refuelling boom without drawing on its safety fuel reserve, the aircraft whose refuelling is divided into two being selected in such a way as to minimise the number of passes at the refuelling boom, the choice initially falling on the aircraft, if it exists, which is upstream in the queue and whose division of refuelling into two passes at the refuelling boom most closely approximates, in terms of a higher value, the desired time gain, then on two aircraft, if they exist, placed upstream in the queue, whose refuelling divisions most closely approximate, in terms of a higher value, the desired time gain, and so on, the absence of a solution in the choice of aircraft with divided refuelling resulting in realisation of the impossibility of refuelling the entire fleet, whereas the existence of a solution results in a modified refuelling sequence proposal, and

- consideration of any modified refuelling sequence proposal and testing of its viability by restarting from the second step of the method.

2. Method according to claim 1, characterised in that it comprises an additional stage, in the case of a negative viability test on the arbitrary refuelling sequence initially considered, this stage entailing the replacement of said arbitrary sequence with a "nominal" refuelling sequence defined by queuing the aircraft in the fleet in an order corresponding to the decreasing quantities of fuel required and by a single pass of each aircraft in the fleet (1) at the refuelling boom, the first aircraft in the queue being the aircraft which required the largest quantity of fuel.

3. Method according to claim 1, characterised in that it comprises an additional stage, in the case of a negative viability test on the arbitrary refuelling sequence initially considered, said stage entailing the replacement of said arbitrary sequence with a refuelling sequence defined by queuing the aircraft in the fleet in an order corresponding to increasing maximum distances which can be covered L1, L2...., Ln and with a single pass of each aircraft in the fleet (1) at the refuelling boom, the

aircraft at the front of the queue being the aircraft which can cover the shortest maximum distance.

4. Method according to claim 1, characterised in that, in the case of a negative viability test on a refuelling sequence due to the identification of an aircraft in the fleet (1) which will be too late in starting its refuelling, the choice of the aircraft located upstream in the queue whose refuelling is divided up results from systematic virtual tests carried out going back along the queue, taking account of one aircraft, then two, including the aircraft achieving the greatest time gain, then three, two of which achieve the greatest time gain, and so on.

5. Method according to claim 1, characterised in that the duration of a first shortened pass at the refuelling boom of a divided refuelling operation is fixed at the same value DMIN for all the aircraft in the fleet (1), said duration being gradually increased, once the refuelling sequence under consideration has passed the viability test, until the flying range limit is reached for one of the aircraft in the fleet.

6. Method according to claim 1, characterised in that the choice of the aircraft in the fleet (1) whose refuelling is divided up results from the verification of observance of a succession of inequality relationships between the travelling distances D1,...., Dn required by the refuelling aircraft (2) to deliver the quantities of fuel required during each pass of the aircraft in the fleet (1) at the refuelling boom and the maximum distances L1, L2,..., Ln which can be covered by each aircraft in the fleet (1).

7. Method according to claim 1, applied to a fleet of two aircraft characterised in that it entails:

- assuming an arbitrary queuing order for the two aircraft in the fleet,

- verifying whether the distance L2 which can be covered by the aircraft in second position in the refuelling queue is greater than the duration, expressed as the distance D1 which can be covered and which is necessary

for the refuelling in one pass at the refuelling boom of the aircraft located in first position in the queue,

- if so, the condition $D1<D2$ being satisfied, adopting the refuelling sequence D1, D2 with a single pass by the refuelling aircraft, and

- if not, the condition $D1<D2$ not being satisfied, verifying that the maximum distance L2 which can be covered by the aircraft located in second position in the queue is greater than the minimum duration, expressed as the distance which can be covered DMIN, of one pass at the refuelling boom,

    - if not, the condition $DMIN<L2$ not being satisfied, indicate the impossibility of refuelling with the adopted queuing order, and

    - if so, the condition $D1<L2$ being satisfied, adopting the refuelling sequence D11, D2, D12 with a division into two passes of the refuelling of the aircraft at the front of the queue.

8. Method according to claim 7, characterised in that the duration of the first pass D11 of refuelling of the aircraft in first position in the queue is taken as equal to the maximum distance L2 which can be covered by the aircraft located in second position in the queue.

9. Method according to claim 1, applied to a fleet of three aircraft characterised in that it entails:

- assuming a refuelling queuing order for the three aircraft in the fleet, corresponding to decreasing quantities of fuel required,

- verifying whether the maximum distances L2 and L3 which can be covered by the aircraft located in second and third positions in the queue are greater than the durations D1, D1+D2, expressed as the distances which can be

covered for the refuelling in one pass at the refuelling boom of the aircraft located ahead in the queue,

- if so, the conditions $D1<D2$ and $D1+D2<L3$ being satisfied, adopting the refuelling sequence D1, D2, D3 with a single pass at the refuelling boom by the refuelling aircraft,

- if not, at least one of the conditions $D1<D2$ or $D1+D2<L3$ not being satisfied, verifying that the maximum distances L2, L3 which can be covered by the aircraft not located at the front of the queue are greater than the minimum durations, expressed as the distances which can be covered DMIN, of one pass at the refuelling boom of each of the aircraft located ahead in the queue,

    - if not, one of the conditions $DMIN<L2$ or $2DMIN<L3$ not being satisfied, indicate the impossibility of refuelling with the adopted queuing order,

    - if so, the two conditions $DMIN<L2$ and $2DMIN<L3$ being satisfied, search for the refuelling(s) whose divisions produce a refuelling sequence which makes it possible to start the refuelling of all the aircraft in the fleet before they have travelled the maximum distances that they can cover, while involving only a minimum number of passes at the refuelling boom,

said search for the refuelling(s) which are to be divided in order to be able to start the refuelling of the aircraft before they have travelled the maximum distances which they can cover comprising the following successive steps:

- verifying whether the conditions $D1<L2$ and $D1+DMIN<L3$ are satisfied,

- if so, adopting the refuelling sequence D1, D21, D3, D22 with a single division into two passes of the refuelling of the aircraft located in second position in the queue,

- if not, one of the conditions D1<L2 or D1+DMIN<L3 not being satisfied, verifying whether the condition D2+DMIN<L3 is satisfied,

- if the condition D2+DMIN<L3 is satisfied, adopting the refuelling sequence D11, D2, D3, D12 with a single division into two passes of the refuelling of the aircraft located at the front of the queue, and

- if the condition D2+DMIN<L3 is not satisfied, adopting the refuelling sequence D11, D21, D3, D12 with divisions into two passes of the refuelling of the aircraft located in first and second positions in the queue.

10. Method according to claim 9, characterised in that, on arriving at the selection of the refuelling sequence D1, D21, D3, D22 with a single division into two passes of the refuelling of the aircraft in second position in the queue, the duration of the first refuelling pass of the aircraft in second position in the queue, expressed as the distance D21, is taken as equal to the maximum distance L3 which can be covered by the aircraft located at the front of the queue, minus the distance D1 required for refuelling in a single pass of the aircraft located at the front of the queue:

$$D21=L3-D1$$

the duration of the second refuelling pass of the aircraft located in second position in the queue expressed as the distance D22 being taken as equal to the duration required for the total quantity of fuel transferred to reach the required quantity:

$$D22=D2-D21$$

11. Method according to claim 9, characterised in that, on arriving at the selection of the refuelling sequence D11, D2, D3, D12 with a single division into two passes of the refuelling of the aircraft in first position in the queue, the duration of the first refuelling pass of the aircraft in first position in the queue, expressed as the distance D11, is taken as equal to the lowest of the maximum distance L2 which can be covered by the aircraft located in second position in the queue and the maximum distance L3 which can be covered by the aircraft located in third position in the queue, minus the distance D2 required for refuelling in a single pass of the aircraft located in second position in the queue:

$$D11=\min(L2, L3-D2)$$

the duration of the second refuelling pass of the aircraft located in first position in the queue expressed as the distance D12 being taken as equal to the duration required for the total quantity of fuel transferred to reach the required quantity:

$$D12=D1-D11$$

12. Method according to claim 9, characterised in that, on arriving at the selection of the refuelling sequence D11, D21, D3, D12, D22 with the divisions into two passes of the refuelling of the aircraft in first and second positions in the queue, the durations of the first refuelling passes of the aircraft in first and second positions in the queue, expressed as the distances D11, D21 are taken as equal to the minimum duration DMIN iteratively increased by an increment INC until reaching the limit of observance of the conditions $D11<L2$ and $D11+D21<L3$ imposed by the flying ranges of the aircraft located in second and third positions in the queue, the durations of the second refuelling passes of the aircraft located in first and second positions in the queue, expressed as the distances D12, D22, being taken as equal to the durations required for the total quantities of transferred fuel to reach the required quantities:

$$D12=D1-D11 \text{ and } D22=D2-D21$$

13. Method according to claim 1, applied to a fleet of four aircraft characterised in that it entails:

- assuming a refuelling queuing order for the four aircraft in the fleet, corresponding to decreasing quantities of fuel required,

- verifying whether the maximum distances L2, L3 and L4 which can be covered by the aircraft located in second, third and fourth positions in the queue are greater than the durations D1, D1+D2, D1+D2+D3 expressed as the distances which can be covered for the refuelling in one pass at the refuelling boom of the aircraft located ahead in the queue,

- if so, the conditions $D1<D2$, $D1+D2<L3$ and $D1+D2+D3<L4$ being satisfied, adopting the refuelling sequence D1, D2, D3, D4 with a single pass at the refuelling boom by the refuelling aircraft,

- if not, at least one of the conditions $D1<D2$ or $D1+D2<L3$ or $D1+D2+D3<L4$ not being satisfied, verifying that the maximum distance L2, L3, L4 which can be covered by the aircraft not located at the front of the queue are greater than the minimum durations, expressed as the distance which can be covered DMIN, of one pass at the refuelling boom of each of the aircraft located ahead in the queue,

    - if not, one of the conditions $DMIN<L2$ or $2DMIN<L3$ or $3DMIN<L4$ not being satisfied, indicate the impossibility of refuelling with the adopted queuing order,

    - if so, the three conditions $DMIN<L2$, $2DMIN<L3$ and 3DMIN [*sic*] being satisfied, search for the refuelling(s) whose divisions produce a refuelling sequence which makes it possible to start the refuelling of all the aircraft in the fleet before they have travelled the maximum

distances that they can cover, while involving only a minimum number of passes at the refuelling boom,

said search for the refuelling(s) which are to be divided in order to be able to start the refuelling of the aircraft before they have travelled the maximum distances which they can cover comprising the following successive steps:

- verifying whether the conditions D1<L2 and D1+D2<L3 and D1+D2+DMIN<L4 are satisfied,

- if so, adopting the refuelling sequence D1, D2, D31, D4, D32 with a single division into two passes of the refuelling of the aircraft located in third position in the queue,

- if not, one of the conditions D1<L2 or D1+D2<L3 or D1+D2+Dmin<L4 not being satisfied, verifying whether the conditions D1<L2, D1+DMIN<L3 and D1+D3+DMIN<L4 is satisfied,

- if so, adopting the refuelling sequence D1, D21, D3, D4, D22 with a single division into two passes of the refuelling of the aircraft located in second position in the queue,

- if not, one of the conditions D1<L2 or D1+DMIN<L3 or D1+D3+DMIN<L4 not being satisfied, verifying whether the conditions D2+DMIN<L3 and D2+D3+DMIN<L4 are satisfied,

- if so, adopting the refuelling sequence D11, D2, D3, D12 with a single division into two passes of the refuelling of the aircraft located in first position in the queue,

- if not, one of the conditions D2+DMIN<L3 or D2+D3+DMIN<L4 not being satisfied, verifying whether the conditions D1<L2, D1+DMIN<L3 and D1+2DMIN<L4 are satisfied,

- if so, adopting the refuelling sequence D1, D21, D31, D4, D22, D32 with divisions into two passes of the refuelling of the aircraft located in second and third positions in the queue,

- if not, one of the conditions D1<L2 or D1+DMIN<L3 or D1+2DMIN<L4 not being satisfied, verifying whether the conditions D2+DMIN<L3 and D2+2DMIN<L4 are satisfied,

- if so, adopting the refuelling sequence D11, D2, D31, D4, D12, D32 with divisions into two passes of the refuelling of the aircraft located in first and third positions in the queue,

- if not, one of the conditions D2+DMIN<L3 or D2+2DMIN<L4 not being satisfied, verifying whether the condition D3+2DMIN<L4 is satisfied,

- if the condition D3+2DMIN<L4 is satisfied, adopting the refuelling sequence D11, D21, D3, D4, D12, D22 with divisions into two passes of the refuelling of the aircraft located in first and second positions in the queue,

- if the condition D3+2DMIN<L4 is not satisfied, adopting the refuelling sequence D11, D21, D31, D4, D12, D22, D31 with divisions into two passes of the refuelling of the aircraft located in first, second and third positions in the queue.

14. Method according to claim 13, characterised in that, on arriving at the selection of the refuelling sequence D1, D2, D31, D4, D32 with a single division into two passes of the refuelling of the aircraft in third position in the queue, the duration of the first refuelling pass of the aircraft in third position in the queue, expressed as the

distance D31, is taken as equal to the maximum distance L4 which can be covered by the aircraft located at the end of the queue, minus the distances D1 and D2 required for refuelling in a single pass of the aircraft located in first and second positions in the queue:

$$D31=L4-D1-D2$$

the duration of the second refuelling pass of the aircraft located in third position in the queue expressed as the distance D32 being taken as equal to the duration required for the total quantity of fuel transferred to reach the required quantity:

$$D32=D3-D31$$

15. Method according to claim 13, characterised in that, on arriving at the selection of the refuelling sequence D1, D21, D3, D4, D22 with a single division into two passes of the refuelling of the aircraft located in second position in the queue, the duration of the first refuelling pass of the aircraft located in second position in the queue, expressed as the distance D21, is taken as equal to the lowest of the maximum distance L4 which can be covered by the aircraft located at the end of the queue, minus the distances D1 and D3 required for refuelling in a single pass of the aircraft located in first and third positions in the queue, and the maximum distance L3 which can be covered by the aircraft located in third position in the queue, minus the distance D1 required for refuelling in a single pass of the aircraft located at the front of the queue:

$$D21=\min(L3-D1, L4-D1-D3)$$

the duration of the second refuelling pass of the aircraft located in second position in the queue expressed as the distance D22 being taken as equal to the duration required for the total quantity of fuel transferred to reach the required quantity:

$$D22=D2-D21$$

16. Method according to claim 13, characterised in that, on arriving at the selection of the refuelling sequence D11, D2, D3, D4, D12 with a single division into two passes of the refuelling of the aircraft in first position in the queue, the duration of the first refuelling pass of the aircraft in first position in the queue, expressed as the distance D11, is taken as equal to the lowest of the maximum duration L2 which can be covered by the aircraft located in second position in the queue, the maximum distance L4 which can be covered by the aircraft located at the end of the queue, minus the distances D2 and D3 required for refuelling in a single pass of the aircraft located in second and third positions in the queue, and the maximum distance L3 which can be covered by the aircraft located in third position in the queue, minus the distance D2 required for refuelling in a single pass of the aircraft located in second position in the queue:

$$D11=\min(L2, L3-D2, L4-D2-D3)$$

the duration of the second refuelling pass of the aircraft located in first position in the queue expressed as the distance D12 being taken as equal to the duration required for the total quantity of fuel transferred to reach the required quantity:

$$D12=D1-D11$$

17. Method according to claim 13, characterised in that, on arriving at the selection of the refuelling sequence D1, D21, D31, D4, D22, D32 with the divisions into two passes of the refuelling of the aircraft located in second and third positions in the queue, the durations of the first refuelling passes of the aircraft located in second and third positions in the queue, expressed as the distances D21, D31 are taken as equal to the minimum duration DMIN iteratively increased by an increment INC until reaching the limit of observance of the conditions $D1+D21<L3$ and $D1+D21+D31<L4$ imposed by the flying ranges of the aircraft located in third and fourth positions in the queue, the durations of the second refuelling passes of the aircraft located in second

and third positions in the queue, expressed as the distances D22, D32 being taken as equal to the durations required for the total quantities of transferred fuel to reach the required quantities:

$$D22=D2-D21 \text{ and } D32=D3-D31$$

18. Method according to claim 13, characterised in that, on arriving at the selection of the refuelling sequence D11, D2, D31, D4, D12, D32 with the divisions into two passes of the refuelling of the aircraft located in first and second positions in the queue, the durations of the first refuelling passes of the aircraft in first and third positions in the queue, expressed as the distances D11, D31 are taken as equal to the minimum duration DMIN iteratively increased by an increment INC until reaching the limit of observance of the conditions $D11<L2$, $D11+D2<L3$ and $D11+D2+D31<L4$ imposed by the flying ranges of the aircraft located in second, third and fourth positions in the queue, the durations of the second refuelling passes of the aircraft located in first and third positions in the queue, expressed as the distances D12, D32, being taken as equal to the durations required for the total quantities of transferred fuel to reach the required quantities:

$$D12=D1-D11 \text{ and } D32=D3-D31$$

19. Method according to claim 13, characterised in that, on arriving at the selection of the refuelling sequence D11, D21, D3, D4, D12, D22 with the divisions into two passes of the refuelling of the aircraft located in first and second positions in the queue, the durations of the first refuelling passes of the aircraft located in first and second positions in the queue, expressed as the distances D11, D21 are taken as equal to the minimum duration DMIN iteratively increased by an increment INC until reaching the limit of observance of the conditions $D11<L2$, $D11+D21<L3$ and $D11+D21+D3<L4$ imposed by the flying ranges of the aircraft located in second, third and fourth positions in the queue, the durations of the second refuelling passes of the aircraft located in first and third positions in the queue, expressed as the

distances D12, D32, being taken as equal to the durations required for the total quantities of transferred fuel to reach the required quantities:

$$D12=D1-D11 \text{ and } D22=D2-D21$$

20. Method according to claim 13, characterised in that, on arriving at the selection of the refuelling sequence D11, D21, D31, D4, D12, D21, D32 with the divisions into two passes of the refuelling of the aircraft in first, second and third positions in the queue, the durations of the first refuelling passes of the aircraft in first, second and third positions in the queue, expressed as the distances D11, D21, D31 are taken as equal to the minimum duration DMIN iteratively increased by an increment INC until reaching the limit of observance of the conditions $D11<L2$, $D11+D21<L3$ and $D11+D21+D31<L4$ imposed by the flying ranges of the aircraft located in second, third and fourth positions in the queue, the durations of the second refuelling passes of the aircraft located in first, second and third positions in the queue, expressed as the distances D12, D22, D32 being taken as equal to the durations required for the total quantities of transferred fuel to reach the required quantities:

$$D12=D1-D11,\ D22=D2-D21 \text{ and } D32=D3-D31$$